QUESTION PAPER 2022

Maharashtra Board Class 10 Science and Technology Part I

Solved Previous Year Question Paper -2022

SECTION A

Best Approach to Attempt a Test

➢ Go through all the questions, quickly.

➢ Mark the easy questions you are sure of solving and attempt them first.

➢ Pay attention to keywords. ➢ Solve questions, part by part.

Q1. A.

1. Gold plated ornaments is the example of___.

A. electroplating **B.** alloying **C.** anodising **D.** galvanising

Ans: electroplating

2. The functioning of the satellite launch vehicle is based on __________.

A. Newton's first law of motion
B. Newton's second law of motion
C. Newton's third law of motion
D. Newton's universal law of gravitation

Ans: Newton's third law of motion

3. is one of the combustible components of L.P.G.

A. Ethane **B.** Propane **C.** Methane **D.** Ethene

Ans. Propane

4. The power of a convex lens of the focal length 25 cm is _________.

A. 4.0 D **B.** 0.25 D **C.** -4.0 D **D.** -0.4 D

Ans: 4.0 D

Power = 1/f ; where f is in meters.
= 1/0.25 = 4.0 D

5. colour is deviated the least in the spectrum of white light obtained with a glass prism.

A. Red **B.** Yellow

C. Violet **D.** Blue

Ans: Red

Q1. **B Answer the following:**

1. Find the odd one out:

   A. INSAT B. GSAT

   C. IRS D. PSLV

   Ans: PSLV

2. Complete the correlation: Group 1 : Alkali metals : :_: Halogens

   Ans: Group 17

3.

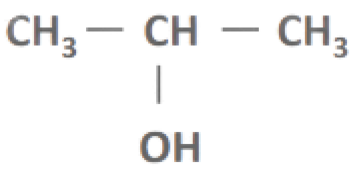
Column 'A'	Column 'B'
Refractive index of water	Refractive index of water a) 1.31 b) 1.36 c) 1.33

Ans: c) 1.33

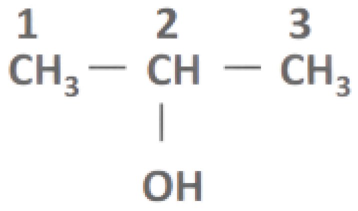
4. **State True or False:**
   An electric motor converts mechanical energy into electrical energy.

   Ans: False

5. **Write the IUPAC name of the following structural formula :**



Solution:



Nomenclature:

Prefix + Word root + Primary suffix + Secondary suffix
Prop ane 2-ol
-IUPAC name: propan-2-ol

Q2. A. Give scientific reasons:

i) Atomic radius goes on increasing down the group.

Solution:

As we move down a group, the atomic number increases causing the number of electrons and shells to increase. This results in an increase in atomic radius down the group.

ii) Simple microscope is used for watch repairs.

Solution:

-A simple microscope has a convex lens that has the ability to produce enlarged as well as erect images of an object.

-Simple microscopes are used by watchmakers to see the small parts and screws of the watch while repairing it.

iii) It is recommended to use airtight container for strong oil for a long time.

Solution:

-Oil, when kept aside for a long time, undergoes oxidation. This causes the oil to develop an unpleasant smell and taste.

-Hence, it is recommended to store oil in air-tight containers to slow down the oxidation reaction.

Q2. B. Answer the following:

i) An object takes 5 s to reach the ground from a height of 5 m on a planet. What is the value of 'g' on the planet?

Ans: Given: $t = 5$ s s

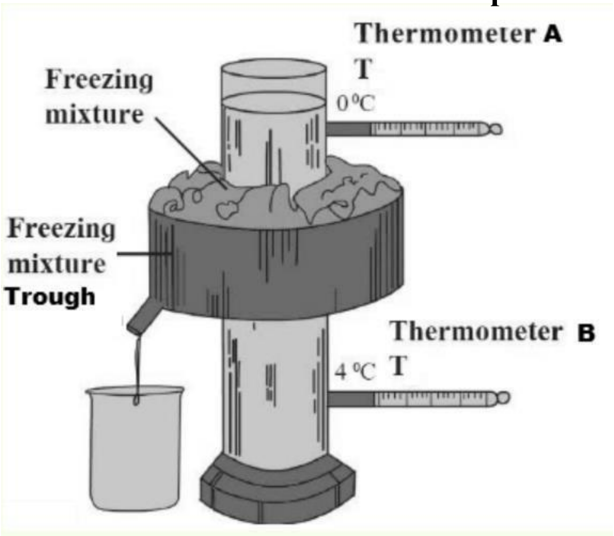
$= 5$ m

$u = 0$ m/s

Using : $s = ut + \frac{1}{2} at^2$ $5 = 0 + \frac{1}{2} g\, 5^2$

Solving $g = 0.4\ m/s^2$

ii) Draw a neat labelled diagram of Hope's Apparatus.



iii) State the laws of refraction

Ans:

a) The incident ray, normal and refracted ray all lie in the same plane at the point of incidence.

b) The ratio of sine of angle of incidence to sine of angle of refraction is constant, for the light of a given colour and for the given pair of media.

iv) a) Name the main ore of aluminium.

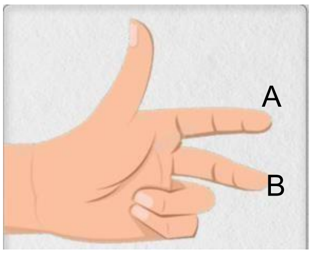
Solution :

The main ore of aluminium is bauxite ($Al_2O_3 . H_2O$).

b) What impurities are present in aluminium ore?

Solution: The ore contains titanium oxide, iron oxide and silicon dioxide as impurities.

v) Observe the given figure of Fleming's Left Hand Rule and write the labels of



'A' and 'B':

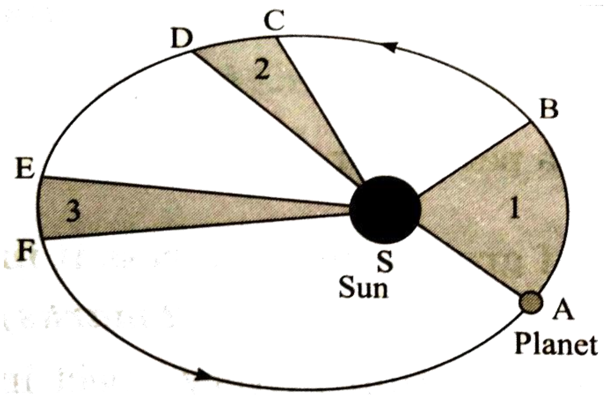
Ans: A: Magnetic Field
B: Current

Q3. Answer the following: A) Write the demerits of Mendeleev's periodic table.

Solution:

1) The position of hydrogen in the periodic table was uncertain.
2) In certain pair of elements, the increasing order of atomic mass is not obeyed. **3)** The periodic table could not explain the position of isotopes.

C) State the laws related to the given diagram :



Ans: Kepler's three laws of planetary motion can be described as follows:

1. The Law of Ellipses: The path of the planets about the sun is elliptical in shape, with the center of the sun being located at one focus.
2. The Law of Equal Areas: An imaginary line drawn from the center of the sun to the center of the planet will sweep out equal areas in equal intervals of time.
3. The ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their average distances from the sun.

D) Identify the type of chemical reaction given below:

i) $CuSO_4 + Fe \rightarrow FeSO_4 + Cu$ **ii)** $2Mg + O_2 \rightarrow 2MgO$ **iii)** $2KClO_3 \rightarrow 2KCl + 3O_2$ t

Solution:

a) $CuSO_4 + Fe \rightarrow FeSO_4 + Cu$: Displacement reaction
b) $2Mg + O_2 \rightarrow 2MgO$: Combination reaction
c) $2KClO_3 \rightarrow 2KCl + 3O_2$ t : Decomposition reaction

E) If the speed of light in a medium is 1.5 x 10^8 m/s, what is the absolute refractive index of the medium? (Speed of light in vacuum = 3 x 10^8 m/s)

Ans:

Refractive index = $\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

= 2

F) Read the following paragraph and answer the question based on it:

If heat is exchanged between a hot and cold object, the temperature of the cold object goes on increasing due to the gain of energy and the temperature of the hot object goes on decreasing due to the loss of energy.

The change in temperature continues till the temperature of both objects attains the same value. In this process, the cold object gains heat energy and the hot object loses heat energy. If the system of both objects is isolated from the environment by keeping it inside a heat resistant box, then no energy can flow from the environment by keeping it inside a heat-resistant box, then no energy can flow from inside the box or come into the box.

a) **Heat is transferred from where to where?**
b) **Which principle do we learn about from this process?**
c) **How will you state the principle briefly?**

Ans:

a) Which principle do we learn about from this process?
Solution: Heat is transferred from a body at a higher temperature to a lower temperature.

c) Which principle do we learn about from this process? Solution: We learn the principle of Heat Transfer.

d) How will you state the principle briefly?
Solution: Heat is a form of energy. Heat always flows from a hot body to a cold body.

F) Complete the following table for convex lens :

S No.	Position of th	Position of th	Nature of the image
1	Beyond		
2		At infinity	
3			Real, inverted & enlarged.

A ns:

S No.	Position of the object	Position of the image	Nature of the image
1	Beyond $2F_1$	Between F_2 & $2F_2$	Real, inverted & diminished.
2	At F_1	At infinity	Real, inverted & highly enlarged.
3	Between F_2 & $2F_2$	Beyond $2F_1$	Real, inverted & enlarged.

G) Explain the following terms:

i) Metallurgy

Solution: The different processes involved in the extraction of metals from their ores and refining are known as metallurgy.

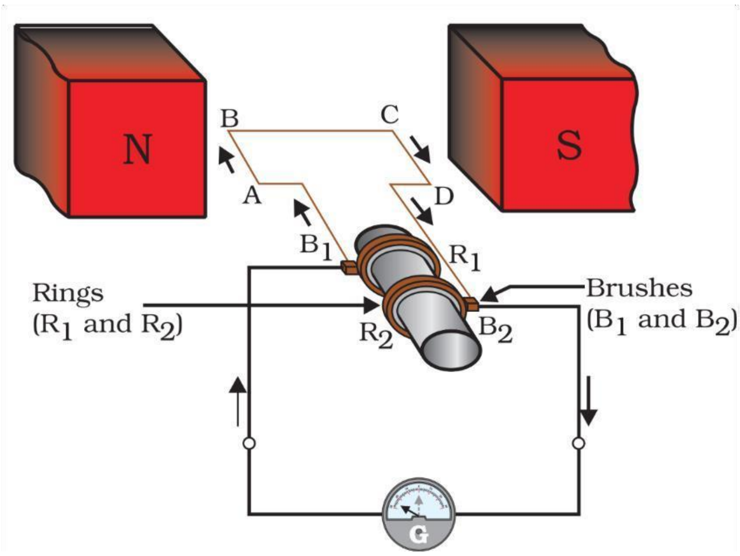
ii) Ores

Solution: The minerals from which the metals can be extracted conveniently and profitably are known as ores.

iii) Gangue

Solution: The unwanted impurities like soil, sand, earthy particles, limestone, rocky material, mica, etc., present in an ore are known as gangue.

Q4. A. Observe the following diagram and answer the questions given below:



b) Identify the above diagram:

Solution: AC Generator

c) State the principle of an electric generator?

Solution: Based on the phenomenon of electromagnetic induction, electric generators are prepared. In an electric generator, mechanical energy is used to rotate a conductor in a magnetic field to produce electricity. This is the principle of an electric generator.

d) Write the working of the above apparatus?

Solution:

i) A rectangular coil that is forced to spin in a uniform magnetic field The coil is connected to a centre-reading meter by metal brushes that press on two metal slip rings (or commutator rings)

ii) The slip rings and brushes provide a continuous connection between the coil and the meter.

iii) When the coil turns in one direction:

• The pointer defects first one way, then the opposite way, and then bac again • This is because the coil cuts through the magnetic field lines and an EMF, and therefore current, is induced in the coil.

iv) The pointer deflects in both directions because the current in the circuit repeatedly changes direction as the coil spins

• This is because the induced EMF in the coil repeatedly changes its direction

• This continues on as long as the coil keeps turning in the same direction

v) The induced EMF and the current alternate because they repeatedly change direction.

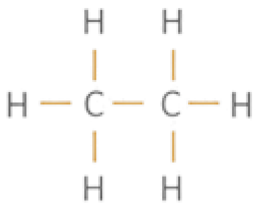
e) Write the use of the above appliance.

Ans: 1. Hydroelectric Power Plant

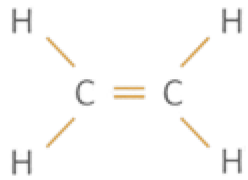
2. Wind Turbines

Q4. B. Identify the saturated and unsaturated hydrocarbon from the given structural formula:

(1)



(2)

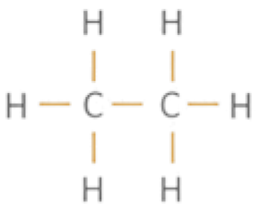


Solution: Here, the hydrocarbons represented in (1) and (2) are ethane and ethene respectively.

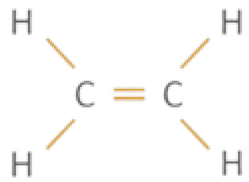
B. Draw electron dot structure for (1) and (2).

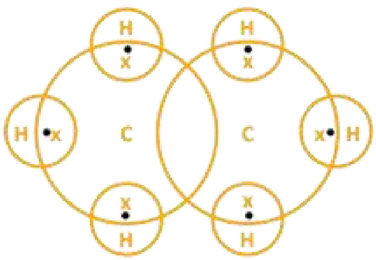
Solution :

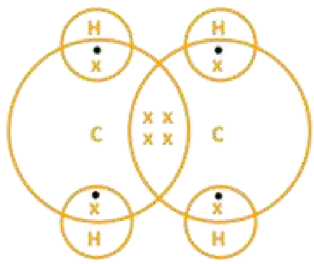
(1)



(2)







C. Define homologous series.

Solution :

- A group of organic compounds with similar structures and chemical properties in which the successive compounds differ by $-CH_2$ group is known as the homologous series.
- For example, methane (CH_4) and ethane (C_2H_6) belong to the same homologous series.

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4. The power of a convex lens of the focal length 25 cm is .

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Ans: Group 17

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Refractive index of water

Refractive index of water

a) 1.31

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Ans: False

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Ans: Given: t = 5 s s = 5 m

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Using : s = ut + ½ at2 5 = 0 + ½ g 52

Ans:

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C) State the laws related to the given diagram :

Ans: Kepler's three laws of planetary motion can be described as follows:

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Solution:

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E) If the speed of light in a medium is 1.5 x 108 m/s, what is the absolute refractive index of the medium? (Speed of light in vacuum = 3 x 108 m/s)

Ans:

Refractive index = **Speed of light in vacuum**

Speed of light in medium

= 2

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Solution: Heat is a form of energy. Heat always flows from a hot body to a cold body.

F) Complete the following table for convex lens :

S

No.

1

Position of the object

Beyond 2F1

Position of the image

Nature of the image

2 At infinity

3 Real, inverted & enlarged.

A ns:

S No.	Position of the object	Position of the image	Nature of the image
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G) Explain the following terms:

i) Metallurgy

Solution: The different processes involved in the extraction of metals from their ores and refining are known as metallurgy.

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Solution: The minerals from which the metals can be extracted conveniently and profitably are known as ores.

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Solution: The unwanted impurities like soil, sand, earthy particles, limestone, rocky material, mica, etc., present in an ore are known as gangue.

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b) Identify the above diagram:

Solution: AC Generator

c) State the principle of an electric generator?

Solution: Based on the phenomenon of electromagnetic induction, electric generators are prepared. In an electric generator, mechanical energy is used to rotate a conductor in a magnetic field to produce electricity. This is the principle of an electric generator.

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Ans: 1. Hydroelectric Power Plant

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Q4. B. Identify the saturated and unsaturated hydrocarbon from the given structural formula:

Solution: Here, the hydrocarbons represented in (1) and (2) are ethane and ethene respectively.

B. Draw electron dot structure for (1) and (2).

Solution :

C. Define homologous series.

Solution :

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- For example, methane (CH_4) and ethane (C_2H_6) belong to the same homologous series.

SSC **MARCH 2022**

MATHEMATICS
ALGEBRA – PART I

Time allowed: 2 hours **Maximum marks: 40**

General Instructions:

(i) All questions are compulsory.
(ii) Use of calculator is not allowed.
(iii) The numbers to the right of the questions indicate full marks.
(iv) In case MCQ's Q. No. 1(A) only the first attempt will be evaluated and will be given credit.
(v) For every MCQ, the correct alternative (A), (B), (C) or (D) of answers with sub question number is:

1. (A) For every sub question four alternative answers are given. Choose the correct answer and write the alphabet of it: **[4]**

(i) Which one is the quadratic equation?

A) $\frac{5}{x} - 3 = x^2$ B) $x(x+5) = 2$

C) $n - 1 = 2n$ D) $\frac{1}{x^2}(x+2) = x$

Answer: B) $x(x+5) = 2$

Solution:

The general form of a quadratic equation is $ax^2 + bx + c = 0$.

Option A: $\frac{5}{x} - 3 = x^2 \Rightarrow x^3 + 3x - 5 = 0$

We can see it is not in the form of $ax^2 + bx + c = 0$. Hence, it is not a quadratic equation.

Option B: $x(x+5) = 2 \Rightarrow x^2 + 5x - 2 = 0$

We can see it is in the form of $ax^2 + bx + c = 0$, with $a = 1$, $b = 5$, and $c = -2$. Hence, it is a quadratic equation.

Option C: $n - 1 = 2n \Rightarrow n + 1 = 0$

We can see it is not in the form of $ax^2 + bx + c = 0$. Hence, it is not a quadratic equation.

Option D: $\frac{1}{x^2}(x+2) = x \Rightarrow x^3 - x - 2 = 0$

We can see it is not in the form of $ax^2 + bx + c = 0$.

Hence, it is not a quadratic equation.

(ii) First four terms of an A.P. are ………, whose first term is – 2 and common difference is -2.

A) $-2, 0, 2, 4$ B) $-2, 4, -8, 16$

C) $-2, -4, -6, -8$ D) $-2, -4, -8, -16$

Answer: C) -2, -4, -6, -8 Solution:

Let the first four terms be a, a + d, a + 2d and a + 3d.

Given, first term $a = -2$ and common difference, $d = -2$, then AP would be: a, a + d, a + 2d and a + 3d

$\Rightarrow -2, -2 + (-2), 2 + 2 \times (-2), 2 + 3(-2)$

$\Rightarrow -2, -4, -6, -8$

(iii) For simultaneous equations in variable x and y, $D_x = 49$, $D_y = -63$, **and D = 7, then what is the value of y?**

A) 9 B) 7

C) -7 D) -9

Answer: D) -9 Solution:

Given, $D_y = -63$, and $D = 7$ We know that,

$$y = \frac{D_y}{D} = \frac{-63}{7} = -9$$

(iv) Which number cannot represent probability?

A) 1.5 B) $\frac{2}{3}$

C) 15% D) 0.7

Answer: A) 1.5 Solution:

$\frac{2}{3} = 0.67$, and $15\% = 0.15$

We know that $0 \leq$ Probability of an event ≤ 1.

So, among 1.5, 0.67, 0.15, and 0.7, 1.5 cannot represent probability.

(B) Solve the following subquestions: **[4]**

(i) To draw a graph of 4x + 5y = 19, find y when x = 1.

Solution:

Given,

Equation of the graph $4x + 5y = 19$

Considering the value of x to be 1,

$\Rightarrow 4 \times 1 + 5y = 19$

$\Rightarrow 4 + 5y = 19$

$\Rightarrow 5y = 19 - 4$

$\Rightarrow y = \frac{15}{5} = 3$

$\therefore y = 3$

(ii) Determine whether 2 is a root of quadratic equation $2m^2 - 5m = 0$.

Solution:
Given quadratic equation,
$2m^2 - 5m = 0$
Substituting $m = 2$, in $2m^2 - 5m = 0$
$\Rightarrow 2(2)^2 - 5(2) = 0$
$\Rightarrow 8 - 10 = 0$
$\Rightarrow -2 \neq 0$
∴ We can observe 2 is not a root of the equation.

(iii) Write second and third term of an A.P. whose first term is 6 and common difference is −3.

Solution:
Given,
First term, a = 6
Common difference, d = −3
We know that, Second term = a + d = 6 + −3 = 6 − 3 = 3
Third term = $a + 2d$ = 6 + 2 × −3 = 6 + (– 6) = 6 − 6 = 0 So,
Second term = 3
Third term = 0

(iv) Two coins are tossed simultaneously. Write the sample space 'S'.

Solution:
Since 2 coins are tossed the sample space ∴ S =
{HH,HT, TH, TT}

2. (A) Complete and write any two activities from the following: [4]

(i) Complete the activity to find the value of the determinant.

$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$

$= 2\sqrt{3} \times$ ____ $- 9 \times$ ___
= ___ − 18
= 0

Solution:
Activity:

$\begin{vmatrix} - & \\ & - \end{vmatrix}$

$= 2\sqrt{3} \times 3\sqrt{3} - 9 \times 2$

= 18 -18
= 0

(ii) Complete the activity to find the $19th$ term of an A.P.: 7, 13, 19, 25.

Activity:
Given A.P.: 7, 13, 19, 25, ………………. Here first term
$a = 7; t_{19} = ?$
$t_n = a + (\_\_\_\_\_\_)d$………………(formula)
$\therefore t_{19} = 7 + (19 - 1)\_\_$
$\therefore t_{19} = 7 + \_\_\_\_\_$
$\therefore t_{19} = \_\_\_\_\_$

Solution: Activity:
Given A.P.: 7, 13, 19, 25, ………………. Here first term
$a = 7; t_{19} = ?$
$t_n = a + (n - 1)d$………………(formula)
$\therefore t_{19} = 7 + (19 - 1)6$
$\therefore t_{19} = 7 + 108$
$\therefore t_{19} = 115$

(iii) If one die is rolled, then to find the probability of an event to get prime number on upper face, complete the following activity.

Activity:
One die is rolled.
'S' is the sample space.
S = { ______________ }
$\therefore n(S) = 6$
Event A : Prime number on the upper face.
A = { ________ }
$\therefore n(A) = 3$

$P(A) = \frac{\_\_}{n(S)}$ …………(formula)

$\therefore P(A) = \_\_\_$

Solution:
Activity:
One die is rolled.
'S' is the sample space.
S = { 1, 2, 3, 4, 5, 6 }
$\therefore n(S) = 6$
Event A : Prime number on the upper face.
A = { 2, 3, 5 }
$\therefore n(A) = 3$

$P(A) = \frac{3}{n(S)}$............(formula)

$\therefore P(A) = \frac{1}{2}$

(B) Solve any four subquestions from the following: [8]

(i) To solve the following simultaneous equations by Cramer's rule, find the values of D_x and D_y.

$3x + 5y = 26$, $x + 5y = 22$

Solution:

By Cramer's rule.

D

$\begin{vmatrix} 2\ 3 & 9 \\ 2\ 3\sqrt{3} & - \end{vmatrix}$

$= 26 \times 5 - 22 \times 5 = 130 - 110 = 20$

$D_y =$

$\begin{vmatrix} 2\sqrt{3} & 9 \\ 2\ 3\sqrt{3} & - \end{vmatrix}$

$= 3 \times 22 - 1 \times 26 = 66 - 26 = 40$

(ii) A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue?

Solution:

Total number of pens = 5 + 8 + 3 = 16

So, the sample space, n(S) = 16

Let A be the event Rutuja picks a blue pen.

Number of blue pens = 8

So, the number of favourable outcomes, (A) = 8

Probability of the pen picked randomly to be blue,

$P(A) = \frac{n(S)}{n(A)} = \frac{8}{16} = \frac{1}{2}$

(iii) Find the sum of first 'n' even natural numbers.

Solution:

First n even natural numbers are 2, 4, 6, ………., 2n.

$t_1 = first\ term = 2$

$t_n = last\ term = 2n$

$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2 + 2n)$

$= \frac{n}{2} \times 2 \times (1 + n)$

$= n \times (1 + n)$

(iv) Solve the following quadratic equation by factorisation method:
$x^2 + x - 20 = 0$

Solution:
$x^2 + x - 20 = 0$
$\Rightarrow x^2 + 5x - 4x - 20 = 0$
$\Rightarrow x(x + 5) - 4(x + 5) = 0$
$\Rightarrow (x - 4)(x + 5) = 0$
$\Rightarrow (x - 4) = 0, (x + 5) = 0$
$\therefore x = 4, x = -5$

(v) Find the values of $(x + y)$ and $(x - y)$ of the following simultaneous equations:

$-8x + y = -80$
$(x + y) = \frac{-80}{-8}$
$\therefore x + y = 10$

$\mathbf{49x - 57y = 172, 57x - 49y = 252}$

Solution:
Adding the given equations, we get

$106x - 106y = 424$
$106(x - y) = 424$
$(x + y) = \frac{424}{106}$
$\therefore x - y = 4$

Subtracting the given equations, we get
$-8x - 8y = -80$

3. (A) Complete the following activity and rewrite it (any one): [3]

(i) One of the roots of equation $kx^2 - 10x + 3 = 0$ is 3. Complete the following activity to find the value of k:

Activity:
One of the roots of equation $kx^2 - 10x + 3 = 0$ is **3**
Putting $x =$ __ in the above equation
$\therefore k(\_\_)^2 - 10 \times \_\_ + 3 = 0$
$\therefore$ ___ $- 30 + 3 = 0$
$\therefore 9k =$ ___
$\therefore k =$ ___

Solution: Activity:
One of the roots of equation $kx^2 - 10x + 3 = 0$ is **3** Putting $x = 3$ in the above equation

$\therefore k(3)^2 - 10 \times 3 + 3 = 0$

$\therefore$ 9k − 30 + 3 = 0

$\therefore 9k = 27$

$\therefore k = 3$

(ii) A card is drawn at random from a pack of well shuffled 52 playing cards. Complete the following activity to find the probability tha the card drawn is – Event A: The card drawn is an ace.

Event B: The card drawn is a spade.

Activity:
'S' is the sample space. $\therefore n(S) = 52$

Event A: The card drawn is an ace.

$\therefore n(A) =$ __

$\therefore P(A) =$ ___……….(formula)

$\therefore P(A) = \frac{\_\_}{52}$

$\therefore P(A) = \frac{\_}{13}$

Event B: The card drawn is a spade.

$\therefore n(B) =$ ___

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

$\therefore n(B) = \frac{\_}{4}$

Solution: Activity:
'S' is the sample space.

$\therefore (S) = 52$

Event A: The card drawn is an ace.

$\therefore$ (A) = 4

$\therefore P(A) = \frac{n(A)}{n(S)}$……….(formula)

$\therefore P(A) = \frac{4}{52}$

$\therefore P(A) = \frac{1}{13}$

Event B: The card drawn is a spade.

$\therefore n(B) = 13$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

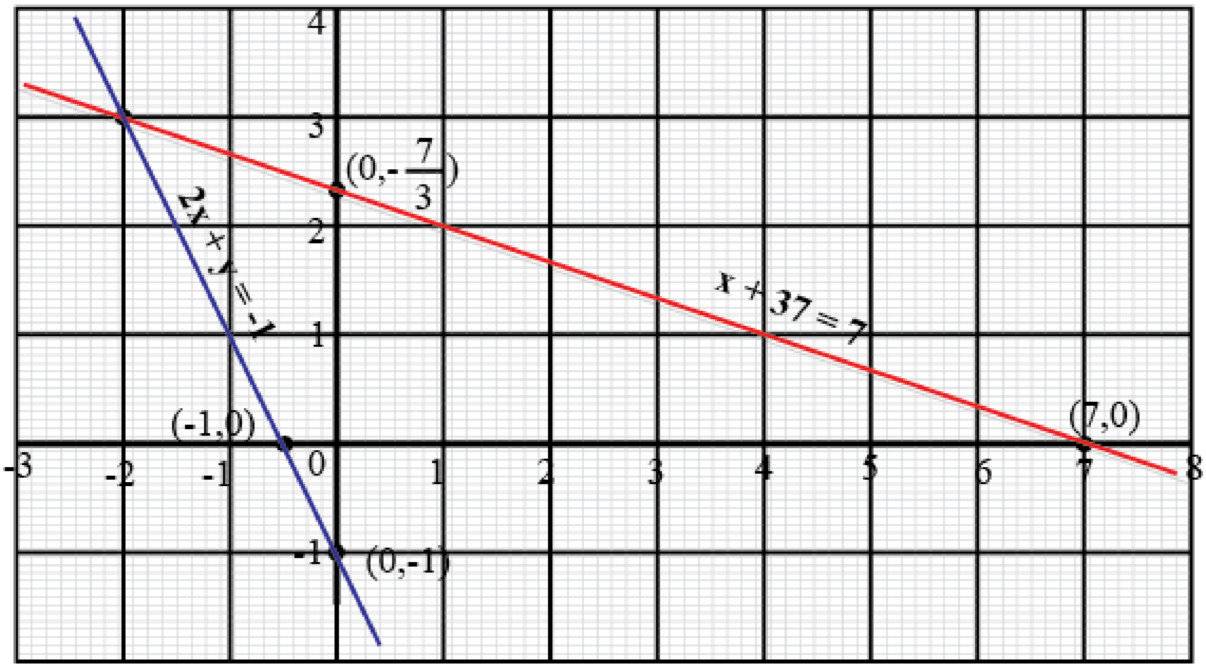
$\therefore n\,(B) = \frac{1}{4}$

(B) Solve the following subquestions (any two): [6]

(i) Solve the simultaneous equations by using graphical method:
$\boldsymbol{x} + \mathbf{3}\boldsymbol{y} = \mathbf{7}$, $\mathbf{2}\boldsymbol{x} + \boldsymbol{y} = -1$

Solution:

Plotting the points (7, 0), $(0, \frac{7}{3})$and $(\frac{-1}{2}, 0)$, $(0, -1)$, we get the graph. (next slide)
We observe that both the graphs are intersecting at −2, 3. ∴ $x = -2$
and $y = 3$ is the solution.



(ii) There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in second row, 4 seats in the third row and so on. Find how many total numbers of seats in the auditorium?

Solution:
The number of seats arranged row-wise is as follows: 20, 22, 24,
….
This sequence is an A. P. with $a = 20$, $d = 22 - 20 = 2$, and $n = 27$

We know, $S_n = \frac{n}{2}(2a + d \times (n - 1))$

$\Rightarrow S_{27} = \frac{27}{2}(2 \times 20 + 2 \times (27 - 1))$

$\Rightarrow S_{27} = \frac{27}{2}(40 + 52)$

$\Rightarrow S_{27} = \frac{27}{2}(92) = 1242$
Total seats in the auditorium are 1242.

(iii) Sum of the present ages of Manish and Savitha is 31 years. Manish's age 3 years ago was 4 times the age of Savitha at that time. Find their present ages.

Solution:

Suppose the present age of Manish is x years and Savitha be y years. According to the first condition, the sum of their present ages is 31.

So, $x + y = 31 \ldots$ (i)

Three years ago;

Age of Manish = $x - 3$ years

Age of Savitha = $y - 3$ years

∴ According to the second condition, 3 years ago Manish's age was 4 times the age of Savitha's. So, $x - 3 = 4\ y - 3$.

$x - 3 = 4y - 12$

∴ $x - 4y = -9$

$x - 4y = -9 \ldots \ldots$ (ii)

Subtracting equation (ii) from (i), We get

$5y = 40$

$\Rightarrow y = 8$

Substituting $y = 8$ equation (i), We get

$x + y = 31$

$x + 8 = 31$

$\Rightarrow x = 23$

Therefore, present age of Manish is 23 years and Savitha is 8 years.

(iv) Solve the following quadratic equation using formula:

$\boldsymbol{x^2 + 10x + 2 = 0}$

Solution:

Comparing the given equation $x^2 + 10x + 2 = 0$ with $ax^2 + bx + c = 0$

∴ We get, $a = 1$, $b = 10$, $c = 2$

We know the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$

$\Rightarrow x = \frac{-10 \pm \sqrt{100 - 4 \times 2}}{2}$

$\Rightarrow x = \frac{-10 \pm \sqrt{92}}{2}$

$\Rightarrow x = \frac{-10 \pm \sqrt{23 \times 4}}{2}$

$\Rightarrow x = \frac{-5 \times 2 \pm 2\sqrt{23}}{2}$

$\Rightarrow x = -5 \pm \sqrt{23}$

∴ Roots of the quadratic equation are $-5 + \sqrt{23}\ and - 5 - \sqrt{23}$.

4. Solve the following subquestions (any two): [8]

(i) If 460 is a natural number, then quotient is 2 more than nines times the divisor and remainder is 5. Find the quotient and divisor

Solution:
Let the divisor be x.
Them, according to question quotient = $9x + 2$
We know, Dividend = Divisor × Quotient + Remainder
$\Rightarrow 460 = x \times (9x + 2) + 5$
$\Rightarrow 460 = 9x^2 + 2x + 5$
$\Rightarrow 455 = 9x^2 + 2x$
$\Rightarrow 9x^2 + 2x - 4500 = 0$
$\Rightarrow 9x^2 + 65x - 63x - 455 = 0$
$\Rightarrow 9(x - 7) + 65\ (x - 7) = 0$
$\Rightarrow (9x + 65)\ (x - 7) = 0$
$\Rightarrow (9x + 65) = 0$ or $(x - 7) = 0$
$\Rightarrow x = \frac{-65}{9}\ or\ x = 7$
However, 460 is divided by a natural number so $x = 7$.
∴ Divisor = 7
And quotient = 9(7) + 2 = 65.

(ii) If the $9th$ term of an A. P. is zero, then prove that the $29th$ term is double the $19th$ term.

Solution:
We know, n
th term of a sequence is $tn = a + (n - 1)$d
∴ t_9 = ninth term = $a + 9 - 1\ d = a + 8d = 0$ (Given) And t_{29} = 29th
term = $a + 29 - 1\ d = a + 28$d
= $a + 8d + 20$d = 0 + 20d = 20d (∵ $a + 8d = 0$)
$\Rightarrow t_{29} = 20d$

And t_{19} = 19th term = $a + 19 - 1\ d = a + 18$d
= $a + 8d + 10$d = 0 + 10d = 10d ($a + 8d = 0$)
$\Rightarrow t_{19} = 10d$

So, we have, $t_{29} = 20d\ and\ t_{19} = 10d$
Observe that

$$t_{29} = 2 \times t_{19} as\ 20d = 2 \times 10d$$

Hence proved that if the $9th$ term of an $A.\ P.$ is zero, then prove that the $29th$ term is double the $19th$ term.

(iii) The perimeter of an isosceles triangle is $24\ cm$. The length of its congruent sides is $13\ cm$ less than twice the length of its base. Find the lengths of all sides of the triangle.

Solution:
Let the length of the base of isosceles triangle = $x\ cm$

Length of congruent sides = $2x - 13$ cm (Given)
Perimeter of isosceles triangle = 24 cm (Given)

Perimeter = Length of base + length of congruent sides
$\Rightarrow 24 = x + 2x - 13 + 2x - 13$
$\Rightarrow 24 = 5x - 26$
$\Rightarrow 50 = 5x$

$\Rightarrow x = 10$
So, the length of Base = 10 cm
Congruent side = $2x - 13 = 20 - 13 = 7$

$\therefore$ The length of base is 10 cm and the length of congruent side are 7 cm and 7 cm.

5. Solve the following subquestions (any one): [3]

(i) A bag contains 8 red and some blue balls. One ball is drawn at random from the bag. If ratio of probability of getting red ball and blue ball is 2:5, then find the number of blue balls.

Solution:
Suppose the number of blue balls = x
$\Rightarrow$ (Blue ball) = x
Number of red balls = 8
$\Rightarrow$ (Red ball) = 8
Total number of balls = $8 + x$
$\Rightarrow$ (Total) = $8 + x$

$$\therefore P(\text{Blue ball drawn}) = \frac{n(\text{Blue ball})}{n(\text{Total})} = \frac{x}{8+x}$$

According to the given condition,

$$\frac{P(\text{Blue ball drawn})}{P(\text{Red ball drawn})} = \frac{5}{2}$$

$$\frac{P(\text{Blue ball drawn})}{P(\text{Red ball drawn})} = \frac{\frac{x}{(8+x)}}{\frac{8}{(8+x)}} = \frac{x}{8}$$

$$\Rightarrow \frac{x}{8} = \frac{5}{2}$$

$\Rightarrow x = 20$
Hence, the number of blue balls is 20.

(ii) Measures of angles of a triangle are in $A.P$. The measure of smallest angle is five times of common difference. Find the measures of all angles of a triangle. (Assume the measures of angles as a, $a + d$, $a + 2d$.)

Solution:

Let the angles of triangles be a, $a + d$, $a + 2d$.

We know that, sum of angles of a triangle = 180°

$\Rightarrow a + a + d + a + 2d = 180°$

$\Rightarrow 3a + 3d = 180°$

$\Rightarrow a + d = 60°$

According to the given conditions,

Smallest angle, $a = 5d$

Putting $a = 5d$ in $a + d = 60°$

$\Rightarrow 6d = 60°$

$\Rightarrow d = 10°$

Putting $d = 10°$ in $a + d = 60°$

$\Rightarrow a + 10 = 60°$

$\Rightarrow a = 50°$

As, angles of triangles are a, $a + d$, $a + 2d$,

Hence, $a = 50°$ And $a + d =$
$60°$

And $a + 2d = 70°$.

∴ Angles of the given triangle are 50°, 60°, and 70°.

SSC **MARCH 2022**

MATHEMATICS
GEOMETRY – PART II

Time allowed: 2 hours **Maximum marks: 40**

General Instructions:

(i) All questions are compulsory.
(ii) Use of calculator is not allowed.
(iii) The numbers to the right of the questions indicate full marks.
(iv) In case MCQ's Q. No. 1(A) only the first attempt will be evaluated and will be given credit.
(v) For every MCQ, the correct alternative (A), (B), (C) or (D) of answers with sub question number is:

1. (A) For every sub question four alternative answers are given. Choose the correct answer and write the alphabet of it: **[4]**

(i) If $\Delta ABC \sim \Delta DEF$ and $\angle A = 48°$, then $\angle D$ = ___________________.

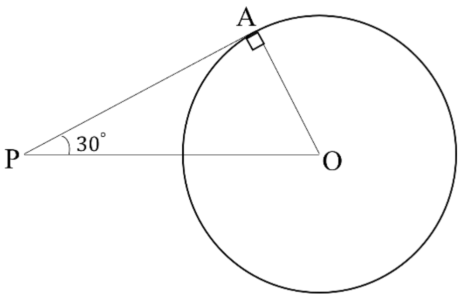
A) 48°　　B) 83°
C) 49°　　D) 132°

Sol: If $\Delta ABC \sim \Delta DEF$ *and* $\angle A = 48°$, then $\angle D = 48°$.
(The corresponding angles in a triangle have the same measure.)

(ii) AP is a tangent at A drawn to the circle with centre O from an external point P. OP = 12 cm and $\angle OPA = 30°$, then the radius of a circle is ___________.

A) $12\ cm$　　B) $6\sqrt{3}\ cm$
C) $6\ cm$　　D) $12\sqrt{3}\ cm$

Sol: Give OP = 12 cm, $\angle OPA = 30°$
As tangent will be perpendicular to radius of the circle
So, $\angle OPA = 90°$
In ΔAPO, $\angle OAP = 90°$



$\therefore \sin 30° = \frac{OA}{OP}$

$\Rightarrow \frac{1}{2} = \frac{OA}{12} \Rightarrow OA = 6\ cm$

(iii) Seg AB is parallel to X – axis and co – ordinates of the point A are (1, 3), then the coordinates of the power B can be __________.

A) (-3, 1) (B) (5, 1) (C) (3, 0) (D) (-5, 3)

Sol: Co – ordinates of point A are (1, 3), then the co – ordinates of the point B can be (-5, 3) as y co – ordinate should be same if seg AB is parallel to X – axis.

(iv) The value of 2tan 45° − 2 sin 30° is ____________.

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Sol: We know that $\tan 45^\circ = 1$ and $\sin 30^\circ = \frac{1}{2}$

Thus, we get $2 \tan 45^\circ = 2 \sin 30^\circ = 2 \times 1 - 2 \times \frac{1}{2}$

$= 2 - 1 = 1$

1. (B)

(i) In $\Delta ABC, \angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ$. If AC = $9\sqrt{2}$, then find the value of AB.

Sol: Given, in ΔABC

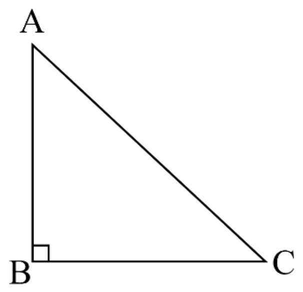
$\angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ$, AC = $9\sqrt{2}$

Now, AB = $\frac{1}{\sqrt{2}} \times AC$

$[Property\ of\ 45^\circ - 45^\circ - 90^\circ\ triangle]$

$\therefore AB = \frac{1}{\sqrt{2}} \times 9\sqrt{2}$

$\therefore AB = 9$



(ii) Chord AB and chord CD of a circle with centre O are congruent. If m (arc AB) = 120°, then find the m (arc CD.)

Sol: Given, chord AB = Chord CD m (arc AB) = 120° We know that,

Arc AB ≅ arc CD

[Corresponding arcs of congruent chord of a circle are congruent]

⟹ m (arc AB) = m (arc CD)

⟹ 120° = m (arc CD)

$\therefore$ m (arc CD) = 120°

(iii) Find the Y-coordinate of the centroid of a triangle whose vertices are (4, -3), (7, 5), and (-2, 1).

Sol: Vertices of the triangle,

(4, - 3), (7, 5), and (-2, 1) ……. [Given]

$x_1 = 4, x_2 = 7, x_3 = -2 \quad y_1 = -3, y_2 = 5, y_3 = 1$

By using the centroid formula,

$$\text{Co-ordinate of centroid} = \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

$$\text{Now, Y-coordinate of centroid} = \frac{y_1 + y_2 + y_3}{3}$$

$$= \frac{-3+5+1}{3} = \frac{3}{3} = 1$$

$\therefore$ Y − coordinate of centroid = 1.

(iv) If sin θ = cos θ, then what will be the measure of angle θ ?

Sol: Given, sin θ = cos θ

We know that, $\sin\theta = \cos(90^\circ - \theta)$ $\therefore$

$\cos\theta = \cos(90^\circ - \theta)$

$\Longrightarrow \theta = 90^\circ - \theta$

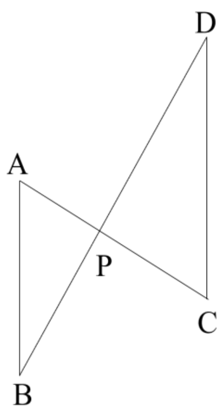
$\Longrightarrow \theta + \theta = 90^\circ \Longrightarrow 2\theta = 90^\circ$

$\Longrightarrow \theta = \frac{90^\circ}{2}$

$\therefore \theta = 45^\circ$

2. (A)

(i) In the given figure, seg AC and seg BD intersect each other in point P. If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove ΔABP ~ ΔCDP.



Sol: Activity: In ΔAPB and ΔCDP

$\frac{AP}{CP} = \frac{BP}{DP}$ ………….. [Given]

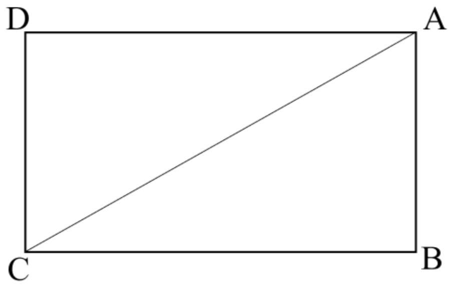
$\therefore \angle APB \cong$ [$\angle DPC$] ……. Vertically opposite angles

$\therefore$ [$\angle ABP$] $\sim \Delta CDP$ …….. test of similarity

(ii) In the given figure, □ABCD is a rectangle. If AB = 5, AC = 13, then complete the following activity to find BC.

Sol: Activity:

ΔAPB is $\boxed{right - angled}$ triangle.



∴ By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\therefore 25 + BC^2 = \boxed{169}$

$\therefore BC^2 = \boxed{144}$

$\therefore BC = \boxed{12}$

(iii) Complete the following activity to prove: cot θ + tan θ = cosec θ × sec θ

Sol: Activity:

L.H.S = Cot $\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\boxed{\sin\theta}}{\cos\theta} = \frac{\boxed{\cos^2\theta} + \sin^2\theta}{\sin\theta \times \cos\theta}$

$= \frac{1}{\sin\theta \times \cos\theta}$ ……… $\boxed{\because \cos^2\theta + \sin^2\theta = 1}$

$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

$= \boxed{cosec\,\theta} \times \sec\theta$

∵ L. H. S = R. H. S

2. (B)

(i) If ΔABC ~ ΔPQR, AB: PQ = 4 : 5 and A (Δ PQR) = 125 cm², then find A (ΔABC).

Sol: Given: ΔABC ~ ΔPQR

We know that,

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$ … … … … [Theorem of area of similar triangles]

$\Rightarrow \frac{A(\Delta ABC)}{125} = \left(\frac{4}{5}\right)^2$

$\Rightarrow \frac{A(\Delta ABC)}{125} = \frac{16}{25}$

$\Rightarrow A(\Delta ABC) = \frac{16}{25} \times 125$

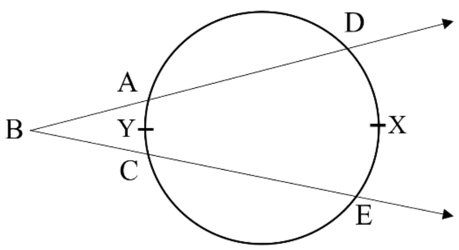
$\therefore A\,(\Delta ABC) = 80\ cm^2$

(ii) In the given figure, m (arc DXE) = 105°, m (arc AYC) = 47° then find the measure of ∠DBE.

Sol: From Figure.

Chord AD and CE intersect externally at point B.

$\therefore m\,(arc\ DEX) = \frac{1}{2}\,[m\,(arc\ DXE) - m(arc\ AYC)]$ …….. [By inscribed angle theorem]



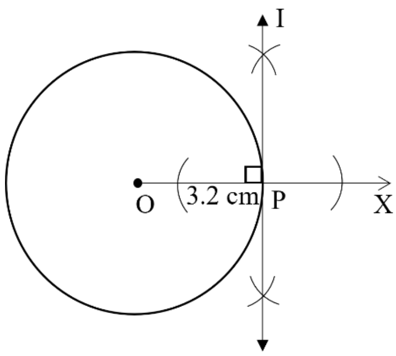
$\therefore m\,(arc\ DEX) = \frac{1}{2}\,[105^\circ - 47^\circ]$

$\therefore m\,(arc\ DEX) = \frac{1}{2}\,[58^\circ]$

$\therefore m\,(arc\ DEX) = 29^\circ$

(iii) Draw a circle of radius 3.2 cm and centre O. Take any point P on it. Draw tangent to the circle through Point P using the centre of the circle.

Sol: Given: Radius of the circle = 3.2 cm



Construction:

(i) With O as the centre draw a circle of radius 3.2 cm.

(ii) Take a point P on the circle and draw ray OP.

(iii) Draw line 1 Perpendicular to ray OX through point

(iv) Line 1 is the required tangent to the circle at point P.

(iv) If $\sin\theta = \frac{11}{61}$, then find the value of $\cos\theta$ using trigonometric identity.

Sol: We know $sin^2\theta + cos^2\theta = 1$

$\Rightarrow cos^2\theta = 1 - sin^2\theta$

$\Rightarrow cos^2\theta = 1 - \left(\frac{11}{61}\right)^2$

$\Rightarrow cos^2\theta = 1 - \left(\frac{121}{3721}\right) = \frac{3721-121}{3721} = \frac{3600}{3721}$

$\Rightarrow cos^2\theta = \sqrt{\left(\frac{60}{61}\right)^2} = \frac{60}{61}$

Thus the value of $\cos\theta$ is $\frac{60}{61}$.

(v) In ΔABC, $AB = 9\ cm$, $BC = 40\ cm$, and $AC = 41\ cm$. State whether ΔABC is a right – angled triangle or not? Write reason.

Sol: Side of ΔABC are $AB = 9\ cm$, $BC = 40\ cm$, $AC = 41\ cm$

The triangle's longest side measures 41 cm.

$\therefore 41^2 = 1681$ …….. (i)

Now, the sum of the square of the remaining sides is

$9^2 + 40^2 = 81 + 1600$

$= 1681$ …….. (ii)

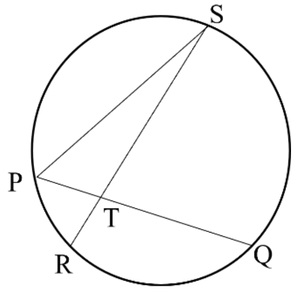
From equations (i) and (ii), as the square of the longest side equals the sum of the squares of the remaining two sides, by suing converse of Pythagoras theorem the given sides from a right – angle triangle.

3. (A)

(i) In the given figure, chord PQ and chord RS intersect each other at point T. If $\angle STQ = 58^\circ$ and $\angle PSR = 24^\circ$, then complete the following activity to verify:

$$\angle STQ = \frac{1}{2}[m(\text{arc } PR) + m(\text{arc } SQ)]$$

Sol: Activity:



In ΔPTS,

$\angle SPQ = \angle STQ - \boxed{\angle PSR}$ $\quad \because$ Exterior angle theorem

$\angle SPQ = 34^\circ$

$\therefore m(arc\ QS) = 2 \times \boxed{34}^\circ = 68^\circ$ ..... $\boxed{\because \text{Inscribed angle theorem}}$

Similarly m (arc PR) = 2 $\angle PSR = \boxed{48}^\circ$

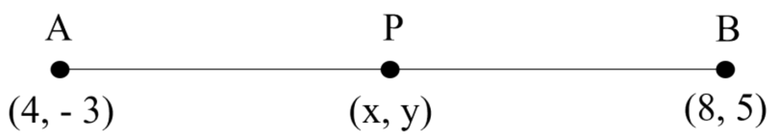
$\therefore \frac{1}{2}[m(arc\ QS) + m(arc\ PR)] = \frac{1}{2} \times \boxed{116}^\circ = 58^\circ$ ....(I)

But $\angle STQ = 58^\circ$ ... . (II)given

$\therefore \frac{1}{2}[m(\text{arc PR}) + m(\text{arc QS})] = \boxed{\angle STQ}$ ... ... [From (I)and (II)]

(ii) Complete the following activity to find the co – ordinates of point P which divides seg AB in the ratio 3 : 1 where (4, -3) and B (8, 5).

Sol. Activity:



$\therefore$ By section formula,

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore x = \frac{3 \times 8 + 1 \times 4}{3+1}, \quad y = \frac{3 \times 5 + 1 \times (-3)}{3+1}$$

$\therefore x = \frac{\boxed{24} + 4}{}, \quad y = \frac{\boxed{15} - 3}{},$

4 4

$\therefore x = \boxed{7} \therefore y = \boxed{3}$

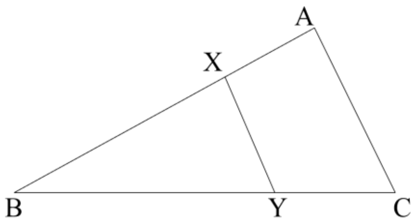
3. (B)

(i) In ΔABC, seg XY ∥ side AC. If 2AX = 3BX and XY = 9, then find the value of AC.

Sol: Given seg XY ∥ seg AC, 2AX = 3BX and XY = 9

Consider, 2AX = 3BX

$\therefore \frac{AX}{BX} = \frac{3}{2}$



$\Rightarrow \frac{AX+BX}{BX} = \frac{3+2}{2}$ …… (By Compenendo)

$\Rightarrow \frac{AB}{BX} = \frac{5}{2}$ ……… (I)

ΔBCA ~ ΔBYX … … … .. SAS test of similarity

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ … …. corresponding sides of similar triangles

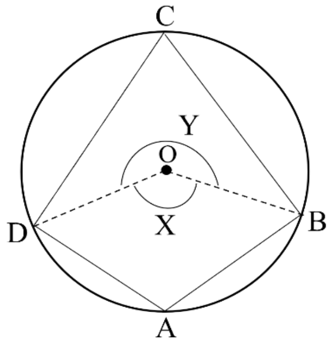
$\therefore \frac{5}{2} = \frac{AC}{9}$ … … … from (I)

$\therefore$ AC = 22.5

(ii) Prove that, "Opposite angles of cyclic quadrilateral are supplementary".

Sol: Let O be the centre of the circle. Join O to B and D.

Let the angle subtended by the minor arc and the major arc at the centre be x and y respectively.



Proof: $x = 2\angle C$ [Angle at centre theorem] …… (i)

and y = $2\angle A$ ……. (ii)

Adding (i) and (ii), we get

$x + y = 2\angle C + 2\angle A$……. (iii)

But, x + y = 360° …….. (iv)

From (iii) and (iv), we get

$$2\angle C + 2\angle A = 360^\circ$$

$$\Rightarrow \angle C + \angle A = 180^\circ$$

But we know that angle sum property of quadrilateral

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow \angle B + \angle D + 180^\circ = 360^\circ$$

$$\Rightarrow \angle B + \angle D = 180^\circ$$

Hence proved that opposite angles of cyclic quadrilateral are supplementary.

(iii) ΔABC ~ ΔPQR. In Δ ABC, AB = 5. 4 cm, BC = 4. 2 cm, AC = 6. 0 cm, and AB : PQ = 3: 2, then construct ΔABC and ΔPQR.

Sol: $\Delta ABC \sim \Delta PQR$ ………[Given]

We know that corresponding sides of triangle which are similar are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2}$$

$\Rightarrow \frac{AB}{PQ} = \frac{3}{2}$ Also, $\frac{BC}{QR} = \frac{3}{2}$ Also, $\frac{AC}{PR} = \frac{3}{2}$

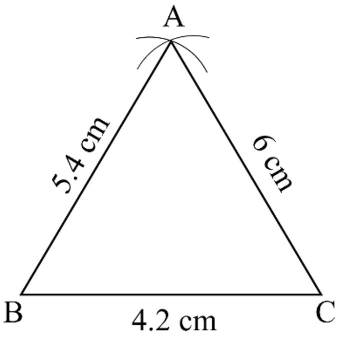
$\Rightarrow \frac{5.4}{PQ} = \frac{3}{2}$ $\Rightarrow \frac{4.2}{QR} = \frac{3}{2}$ $\Rightarrow \frac{6}{PR} = \frac{3}{2}$

$\Rightarrow PQ = \frac{5.4 \times 2}{3} = 3.6\ cm$ $\Rightarrow QR = \frac{4.2 \times 2}{3}$ $\Rightarrow PR = \frac{6 \times 2}{3} = 4cm$

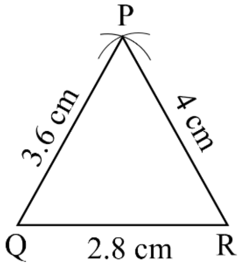
$$\Rightarrow QR = 2.8\ cm$$

Now, draw angle ΔABC with sides AB = 5.4 cm, BC = 4.2 cm and AC = 6 cm.

Also draw triangle ΔPQR with sides PQ = 3.6 cm, QR = 2.8 cm and PR = 4 cm.



[1]



[1]

(iv) Show that: $\left(\frac{\tan A}{1+\tan^2 A}\right)^2 + \left(\frac{\cot A}{1+\cot^2 A}\right)^2 = \sin A \times \cos A$

Sol: $\left(\frac{\tan A}{1+\tan^2 A}\right)^2 + \left(\frac{\cot A}{1+\cot^2 A}\right)^2$

$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\operatorname{cosec}^2 A)^2}$ [$\because 1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \operatorname{cosec}^2 A$]

$= \frac{\sin A}{\cos A} \times \cos^4 A + \frac{\cos A}{\sin A} \times \sin^4 A$

$= \sin A \cos^3 A + \cos A \sin^3 A$

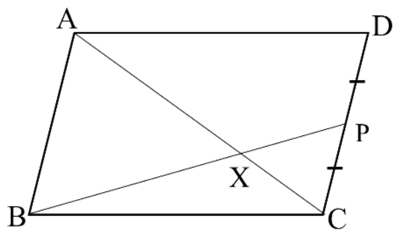
$= \sin A \cos A$

Hence proved.

4.

(i) $\square ABCD$ is parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that: 3 AX = 2AC

Sol: From the figure, in ΔABX and ΔCPX



As, AB ∥ CD

$\angle BAX = \angle PCX$ …… [Alternate angle]

$\angle BXA = \angle PXC$ …… [Vertically opposite angles]

$\therefore \Delta ABX \sim \Delta CPX$ … … . [By AA similarity theorem]

We know that,

Similar triangles have comparable side ratios that are similar to or equal.

$\therefore \frac{AX}{CX} = \frac{AB}{CP}$

But CD = AB and P is mid – point of CD.

$\therefore AB = 2CP$

$\Rightarrow \frac{AX}{AC-AX} = \frac{2CP}{CP} = 2$

$\Rightarrow AX = 2\,(AC - AX)$

$\Rightarrow AX = 2AC - 2AX$

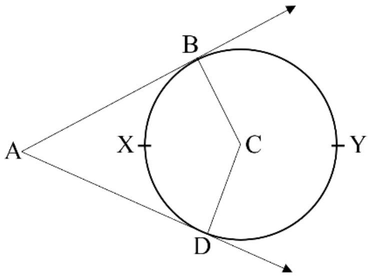
$\Rightarrow AX + 2AX = 2AC$

$\Rightarrow 3AX = 2AC$

Hence proved.

(ii) In the given figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that: $\angle A = \frac{1}{2}\,[m\,(\text{arc BYD}) - m\,(\text{arc BXD})]$

Sol: Proof: From figure



Seg AB ⊥ seg BC and seg AD ⊥ seg CD ……

[By tangent theorem]

$\therefore \angle ABC = \angle ADC = 90^\circ$

In □ABCD,

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ ……. [Angle of the square]

$\therefore \angle A + 90^\circ + \angle C + 90^\circ = 360^\circ$

$\therefore \angle A + \angle C = 360^\circ - 180^\circ$

$\therefore \angle A + \angle C = 180^\circ$

$\therefore \angle A$ + m (arc BXD) = 180° [Central angle] …….. (i)

Now, m (arc BXD) + m (arc BYD) = 360°

…… [Two arcs contribute a complete circle] …….. (ii)

Now, multiply equation (i) by 2 on both sides

$2[\angle A + m\ (arc\ BXD)] = 2 \times 180^\circ$

$\Rightarrow 2\angle A + 2 \times m\ (\text{arc BXD}) = 360^\circ$

$\Rightarrow 2\angle A = 360^\circ - 2 \times m\ (\text{arc BXD})$

$\Rightarrow 2\angle A = m\ (\text{arc BXD}) + m\ (arc\ BYD) - 2m\ (arc\ BXD)$

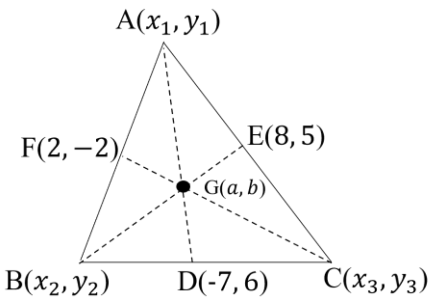
$\Rightarrow 2\angle A = m\ (\text{arc BYD}) - m(\text{arc BXD})$
…. . [From (ii)]

$\Rightarrow \angle A = \frac{1}{2}\ [m(arc\ BYD) - m\ (arc\ BXD)]$

Hence proved.

(iii) Find the co-ordinates of centroid of a triangle if points D (-7, 6), E (8, 5), and F(2, -2) are the mid – points of the sides of the that triangle.

Sol: Suppose A (x_1, y_1), $B\ (x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of triangle. D (-7, 6), E (8, 5) and F (2, - 2) are the midpoints of sides BC, AC and AB respectively. Let G be the centroid of ΔABC. D is the midpoint of seg BC.



By midpoint formula,

Co – ordinates of D = $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

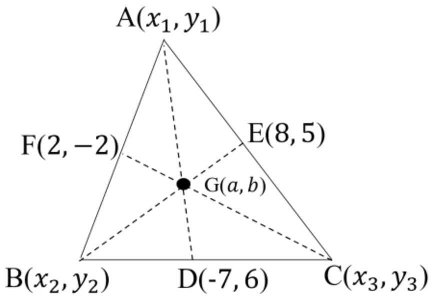
$\Rightarrow (-7, 6) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

$\Rightarrow \frac{x_2 + x_3}{2} = -7, \frac{y_2 + y_3}{2} = 6$

$\Rightarrow x_2 + x_3 = -14$ …. (i), $y_2 + y_3 = 12$ …. (ii),

E is the midpoint of seg AC;

Co – ordinates of E = $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$



$\Longrightarrow (8,5) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$\Longrightarrow \frac{x_1+x_3}{2} = 16, \frac{y_1+y_3}{2} = 10$

$\Longrightarrow x_1 + x_3 = 16 \ldots\ldots$ (iii), $y_1 + y_3 = 10 \ldots$ (iv)

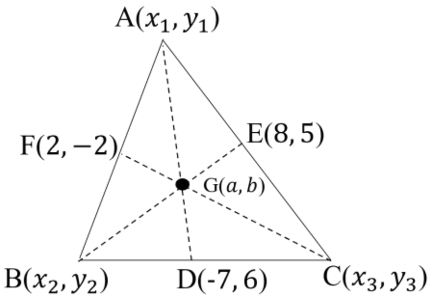
Similarly, as F is the midpoint of seg AB;

Co – ordinates of E = $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$\Longrightarrow x_1 + x_2 = 4 \ldots\ldots$ (v), $y_1 + y_3 = -4 \ldots\ldots$ (vi)

Adding (i), (iii) and (v),

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -14 + 16 + 4$



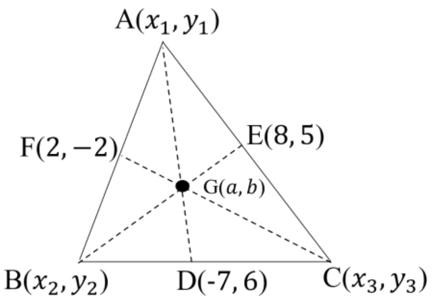
$\Longrightarrow 2x_1 + 2x_2 + 2x_3 = 6 \Longrightarrow x_1 + x_2 + x_3 = 3\ldots$ (vii)

Adding (ii), (iv) and (vi),

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 12 + 10 - 4$

$\Longrightarrow 2y_1 + 2y_2 + 2y_3 = 18 \Longrightarrow y_1 + y_2 + y_3 =$ 9 ... (viii) G is the centroid of ΔABC. By centroid formula,

Co- ordinates of G = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$



$= \left(\frac{3}{3}, \frac{9}{3}\right) \ldots\ldots$ From (vii)and (viii)

= (1, 3)

∴ The Co-ordinates of the centroid of the triangle are (1, 3)

5.

(i) If a and b are natural numbers and a > b If $(a^2 + b^2)$,$(a^2 - b^2)$ and 2ab are the sides of the triangle, then prove that the triangle is right angled. Find out two pythagorean triplets by taking suitable values of a and b.

Sol: $a^2 + b^2$, $a^2 - b^2$, 2ab are sides of triangle.

By Pythagoras' theorem,

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$a^4 + b^4 + 2a^2 b^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$

$a^4 + b^4 + 2a^2 b^2 = a^4 + b^4 + 2a^2b^2$

AS L.H.S. = R.H.S.

∴ Triangle is a right – angle triangle as it follows Pythagorean triplets As $a > b$ ….. [Given]

Let a = 4, b = 3

$a^2 + b^2 = 4^2 + 3^2 = 16 + 9 = 25$

$a^2 - b^2 = 16 - 9 = 7$

$2ab = 2 \times 4 \times 3 = 4$

∴ (25, 7,24) is a Pythagorean triplet.

Let a = 2, b = 1

$a^2 + b^2 = 2^2 + 1^2 = 4 + 1 = 5$

∴ (5, 3, 4) is another Pythagorean triplet.

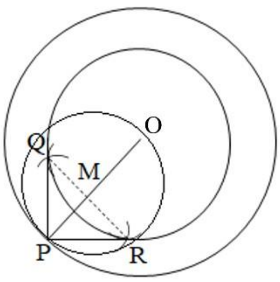
(ii). Construct two concentric circles with centre O with radii 3 cm and 5 cm. construct tangent to a smaller circle from any point A on the larger circle. Measure and write the length of tangent segment. Calculate the length of tangent segment using Pythagoras theorem.

Sol: Following are the steps to draw tangents on the given circle:

Step 1: Draw a circle of 3 cm radius with centre O on the given plane.

Step 2: Draw a circle of 5 cm radius, taking O as its centre.
Locate a point P on this circle and join OP.

Step 3: Bisect OP. Let M be the midpoint of PO.



Step 4: Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at points Q and R.

Step 5: Join PQ and PR. PQ and PR are the required tangents.

It can be observed that PQ and PR are of length 4 cm each.

Since PQ is a tangent,

$\therefore \angle PQO = 90^{\circ}\ and\ PO = 5\ cm\ and\ QO = 3\ cm$

Applying Pythagoras theorem in ΔPQO, we obtain

$PQ^2 + QO^2 = PQ^2$

$\Rightarrow PQ^2 + (3)^2 = (5)^5$

$\Rightarrow PQ^2 + 9 = 25$

$\Rightarrow PQ^2 = 25 - 9 = 16$

$\Rightarrow PQ = 4$ cm